Structures II Review
Sept. 23, 2008

Allowable Stress Design (ASD)

The stresses developed in a structure must not exceed the elastic limit. Factors of safety are used to ensure stress does not exceed the elastic limit.

The ASD approach has generally been replaced internationally by Load and Resistance Factor Design (LRFD).

Load and Resistance Factor Design (LRFD) – Also called Limit State Design or Strength Design

LRFD refers to a design methodology used in structural engineering. The methodology is a modernization and rationalization of engineering knowledge which was well established prior to the adoption of LRFD.

LRFD requires the structure to satisfy two principal limit state criteria:
Note: A limit state is a set of performance criteria (e.g. vibration levels, deflection, strength, stability, buckling, twisting, collapse) that must be met when the structure is subject to loads.

- **The Ultimate Limit State (Strength)**: the structure must not collapse when subjected to the peak design load for which it was designed.
- **The Serviceability Limit State (Serviceability)**: the purpose of serviceability requirements is to ensure that people in the structure are not unnerved by large deflections of the floor, vibration caused by walking, sickened by excessive swaying of the building during high winds, or by a bridge swaying from side to side and to keep beam deflections low enough to ensure that brittle finishes on the ceiling above do not crack, affecting the appearance and longevity of the structure.

Necessary Assumptions:

- The loads must be estimated
- The size of members to check must be chosen
- The design criteria must be selected

Note: The goal of any engineering design is to ensure that a safe and functional structure is created.

Safety Provisions

Structures must always be designed to carry a greater load than what is expected under normal use. This reserve capacity is provided to account for a variety of factors, which can be grouped in two general categories:

1. **Factors relating to overload:** May arise from changing the structures use, underestimation of load effects, effects from construction sequence and methods
2. **Factors relating to understrength:** may result from variations in material strength, workmanship, dimensions, control, or degree of supervision.

The ACI Code strength design method has traditionally divided safety provisions into two parts; U factors to account for the probability of overload, and ϕfactors to account for the probability of understrength. The requirement for strength design may be expressed:

Design Strength ≥ Required Strength (i.e Factored Load)

$$\phi P_n \geq P_u$$

$$\phi M_n \geq M_u$$

$$\phi V_n \geq V_u$$

"Design Strength" is calculated in accordance with the provisions of building code

"Required Strength" is obtained by performing structural analysis using factored loads, also called the ultimate strength.

Where P_n, M_n, V_n are "nominal" or "normal" strengths in axial compression, bending moment, and shear, respectively, using the subscript n. On the load side P_u, M_u, V_u are the factored load effects in axial compression, bending moment, and shear respectively using the subscript u.

For many years the ACI Code has used U and ϕ factors that resulted from combined experience and historical precedent to arrive at the appropriate numerical values.

Overload Factors U

The factor for overload as given by Jo:

$$U = 1.2D + 1.6L$$

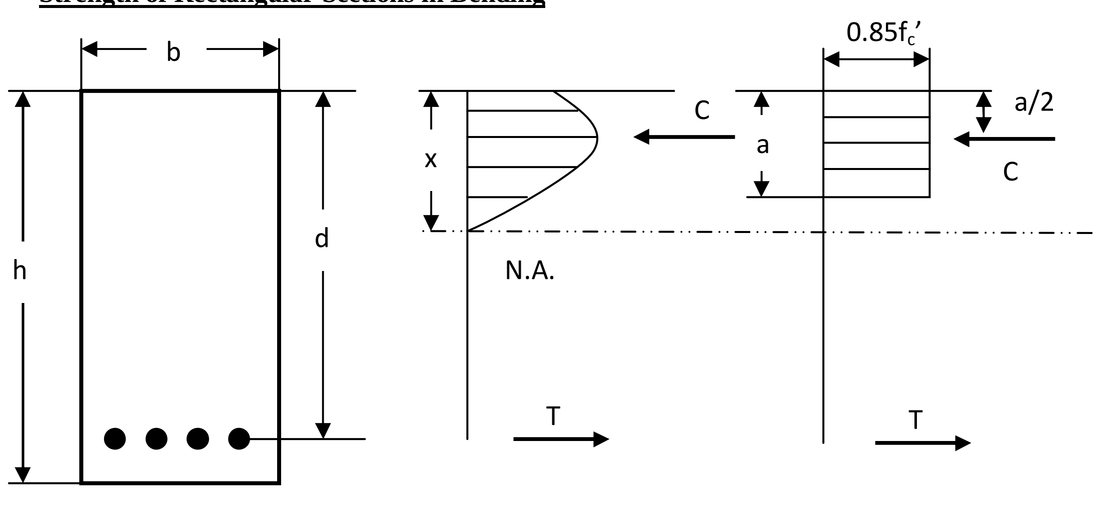
Where D is dead load; L is live load.

Strength Reduction Factors ϕ

The factors ϕ for understrength are called strength reduction factors according to ACI-9.3. These are also called resistance factors in the LRFD for steel structures. The ϕ factors in ACI-9.3 are as follows:

1. Flexure; tension-controlled sections (i.e bending moment (M) and/or axial load (P))
   $\phi = 0.90$
2. Shear and torsion (V, T)
   $\phi = 0.75$ (Jo has mentioned 0.85; this is from ACI-C.2 in use since 1971 until 1999)

Strength of Rectangular Sections in Bending



(a) Beam (b) Actual stress distribution (c) Whitney rectangular stress block

The Compression resultant C is the summation of the compressive stresses acting on the compression concrete area.

$$C = 0.85 f_c' ba$$

Where 0.85 represents the average stress based on the Whitney rectangular stress block, f_c' is the compressive strength of concrete, b is the width of the section, a is the depth of Whitney rectangular stress distribution from the compression face of the beam.

For the ductile failure condition, the tensile force T is:

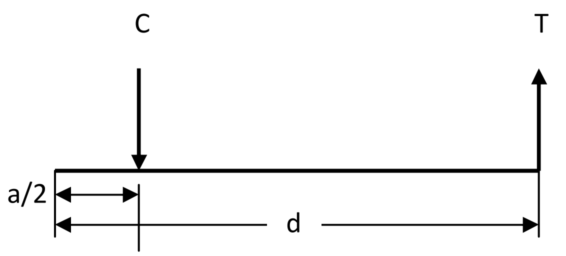
$$T = A_s f_y$$

Equilibrium requires C=T. Therefore you can set C=T and solve for a

$$0.85 f_c' ba = A_s f_y$$

$$a = \frac{A_s f_y}{0.85 f_c' b}$$

Find the Moment; M = Force (Distance)



Find the Moment around T:

$$M_n = C\left(d - \frac{a}{2}\right)$$

$$M_n = 0.85 f_c' ba\left(d - \frac{a}{2}\right)$$

Find the Moment around C:

$$M_n = T\left(d - \frac{a}{2}\right)$$

$$M_n = A_s f_y\left(d - \frac{a}{2}\right)$$

Note: Instead of plugging in the equation for *a*, you can use the approximation method mentioned in class.

$$\left(d - \frac{a}{2}\right) = 0.9d$$

Reinforcement ratio at balanced strain condition for rectangular beam having tension reinforcement only.
The symbol ρ (rho), the reinforcement ratio (often called reinforcement percentage), may be conveniently used to represent the relative amount of tension reinforcement in a beam

$$\rho = \frac{A_s}{bd}$$

<u>Jo's Structures Crib Sheet</u>

Choose an effective depth:

d = effective depth

Use 1 inch per 1 foot of span

Example: If you have a span of 28ft what effective depth should you use?

d = 28in

Choose the width of section:

b = width of section

d = effective depth

b $\leq \frac{d}{2}$

Example: If d = 28in what is b?

b =14in

Since common values for width of section are 8in or 12in we will choose 12in

b =12in

Calculate the dead load if given: concrete load $= \mathbf{150}\ \frac{lb}{ft^3}$

Hint: convert to ft

$Dead\ Load\ (i.e\ D) = (150\ lb\ ft^3)(d)(b)$

$D = \left(150\ \frac{lb}{ft^3}\right)(28in)\left(\frac{1ft}{12in}\right)(12in)\left(\frac{1ft}{12in}\right) = 350\ \frac{lb}{ft}$

Pick your materials:

Steel:

$f_y = yeild\ stress\ of\ steel$

$f_y = 40{,}000psi$

$f_y = 50{,}000psi$

$f_y = 60{,}000psi$

Concrete:

$f_c' = compressive\ strength\ of\ concrete$

$f_c' = 3{,}000psi$

$f_c' = 4{,}000psi$

$f_c' = 5{,}000psi$

Use the approximation method mentioned in class for a.

$$\left(d - \frac{a}{2}\right) = 0.9d$$

Find the area of steel.

$$M_n = A_s f_y (0.9d)$$

$$\phi M_n = M_u$$

$$\phi = 0.9\ (because\ M\ is\ flexture)$$

$$M_u = \phi A_s f_y (0.9d)$$

$$A_s = \frac{M_u}{\phi f_y (0.9d)}$$

How much concrete cover?

You should have at least a 1.5 in cap of concrete beyond your reinforcement. If you are using 1in diameter bars then you will need to add 0.5 in onto the 1.5in minimum. Therefore, in this case use 2in of cover.

List of Symbols

$ACI = American\ Concrete\ Institute$

$ACI\ Code = Building\ Code\ REquirements\ for\ Structural\ Concrete$

$U = ACI\ Code\ factors\ for\ safety\ related\ to\ overload; factored\ load, factored\ moment, factored\ shear$

$\phi = ACI\ code\ strength\ reduction\ factors, use\ 0.90\ for\ Flexture(Tension - controlled),$
$use\ 0.75\ or\ 0.85\ for\ Shear\ and\ Torsion$

$P_n = nominal\ axial\ load\ strength$

$P_u = factored\ axial\ force$

$M_n = nominal\ moment\ strength$

$M_u = factored\ moment$

$V_n = nominal\ shear\ strength, V_c + V_s$

$V_c = nominal\ shear\ strength\ attributed\ to\ concrete$

$V_s = nominal\ shear\ strength\ attributed\ to\ shear\ reinforcement$

$V_u = factored\ shear\ force$

$d = effective\ depth; distancefrom\ compression\ face\ to\ centriod\ of\ tension\ steel$

$f_c^{'} = compressive\ strength\ of\ concrete, measured\ 28\ days\ after\ casting;$
$Force\ on\ concrete\ divided\ by\ area\ of\ concrete$

$b = width\ of\ section$

$a = depth\ of\ Whitney\ rectangular\ stress\ distribution\ from\ the\ compression\ face\ of\ the\ beam$

$f_y = yeild\ stress\ of\ steel$

$A_s = area\ of\ main\ tension\ reinforcement(i.e\ steel)$

$\rho = reinforcement\ ratio, \frac{A_s}{bd}, for\ tension\ steel$

$\rho_b = reinforcement\ ratiofor\ the\ balanced\ strain\ condition$

$\rho_{max} = maximum\ reinforcement\ ratio\ permitted\ by\ ACI - 10.3.5$

$\rho_g =\ reinforcement\ ratio\ based\ on\ gross\ concrete\ area,$

$f_v = stress\ in\ the\ shear\ reinforcement$

$R_n =$ coefficient of resistance

Equations:

$$U = 1.2D + 1.6L$$

$$V_u = \frac{W}{2}$$

$$M_u = \frac{wL^2}{8}$$

$$M_n = \frac{M_u}{\phi}$$

$$\phi M_n = M_u$$

$$R_n = \frac{M_n}{bd^2}$$

$$b \geq \frac{d}{3} \text{ and } b \leq \frac{d}{2}$$

$$A_s = \frac{M_u}{\phi f_y (0.9d)}$$

$$\text{Minimum } A_s = \frac{\left(200\frac{lb}{in^2}\right)(bd)}{f_y}$$

$$\text{Maximum } A_s = 0.75\rho_b bd$$