Structures I Review
April 2, 2008

Centroid of a Body

This is calculated using the geometry of the figure. If the body is homogeneous, the center of mass will be at the centroid To calculate the centroid of a geometrically complex cross section, divide the object into known simple geometries and then apply the following formula.

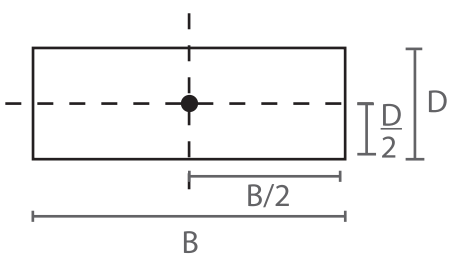
$\overline{x}$ is the distance from your reference coordinate axis to the centroid of the particular area.
A is the area of that particular section.

$$\overline{x} = \frac{\sum \overline{x_i} A_i}{\sum A_i}$$

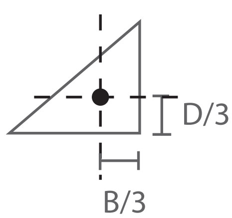
For a three body figure, you may have an equation that looks like this:

$$\overline{x} = \frac{(\overline{x}_1 A_1 + \overline{x}_2 A_2 + \overline{x}_2 A_2)}{(A_1 + A_2 + A_3)}$$

Centroid of a Rectangle



Centroid of a Right Triangle



Moment of inertia

The moment of inertia of an object about a given axis describes how difficult it is to change its angular motion about that axis. For example, consider two discs (A and B) of the same mass. Disc A has a larger radius than disc B. Assuming that there is uniform thickness and mass distribution, it requires more effort to accelerate disc A (change its angular velocity) because its mass is distributed further from its axis of rotation: mass that is further out from that axis must, for a given angular velocity, move more quickly than mass closer in. In this case, disc A has a larger moment of inertia than disc B.

The moment of inertia of an object can change if its shape changes. A figure skater who begins a spin with arms outstretched provides a striking example. By pulling in her arms, she reduces her moment of inertia, causing her to spin faster

Moment of inertia: Rectangle

$$I = \frac{BD^3}{12}$$

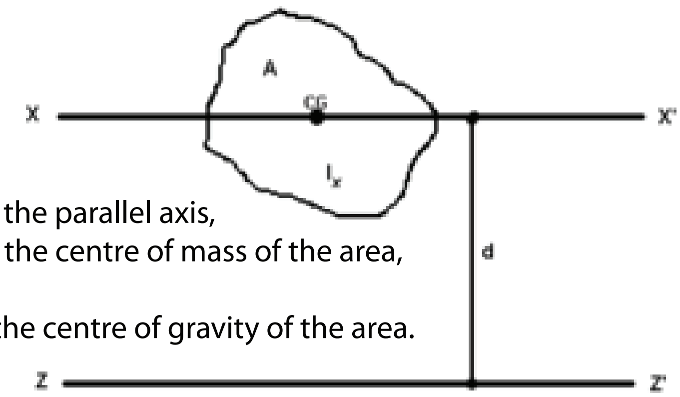
Moment of inertia: Triangle

$$I = \frac{BD^3}{36}$$

Parallel Axis Theorem

the parallel axis theorem can be used to determine the moment of inertia of a rigid object about any axis, given the moment of inertia of the object about the parallel axis through the object's centre of mass and the perpendicular distance between the axes.

$$I_z = I_x + Ad^2.$$



where:
I_z is the area moment of inertia through the parallel axis,
I_x is the area moment of inertia through the centre of mass of the area,
A is the surface of the area, and
d is the distance from the new axis z to the centre of gravity of the area.
($d = \Delta \bar{x}$)

The resultant moment of inertia for a geometricly complex objevt is the sum of the values obtained in the parallel axis theorem.

$$I_z = \sum (I_x + Ad^2)$$

Stress and Strain

Stress is a measure of the average amount of force exerted per unit area

$$\text{Stress } (f) = \frac{\text{Force}}{\text{Area}} = \frac{P}{A}$$

Strain is the geometrical expression of deformation caused by the action of stress on a hysical body. Strain is calculated by first assuming a change between two body states: the beginning state and the final state

$$\text{Strain } (\epsilon) = \frac{\Delta L}{L}$$

Modulus of elasticity is the mathematical description of an object or substance's tendency to be deformed elastically (i.e., non-permanently) when a force is applied to it. The elastic modulus of an object is defined as the slope of its stress-strain curve in the elastic deformation region:

$$\text{Mod of Elasticity } (E) = \frac{\text{Stress}}{\text{Strain}}$$